

Heat-Diffusion Modeling of Event Streams Enables Accurate Frequency Estimation of Periodic Motion

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Executive Summary

Event-based cameras are **asynchronous** sensors that capture only per-pixel intensity changes, defined as an event

$$e = (t, x, y, p)$$

where t = timestamp, x and y are the coordinates, and p is either $\{0, 1\}$ representing polarity. This allows for these cameras to record high-speed motion with microscopic precision, a property we use to analyze **periodic motion**.

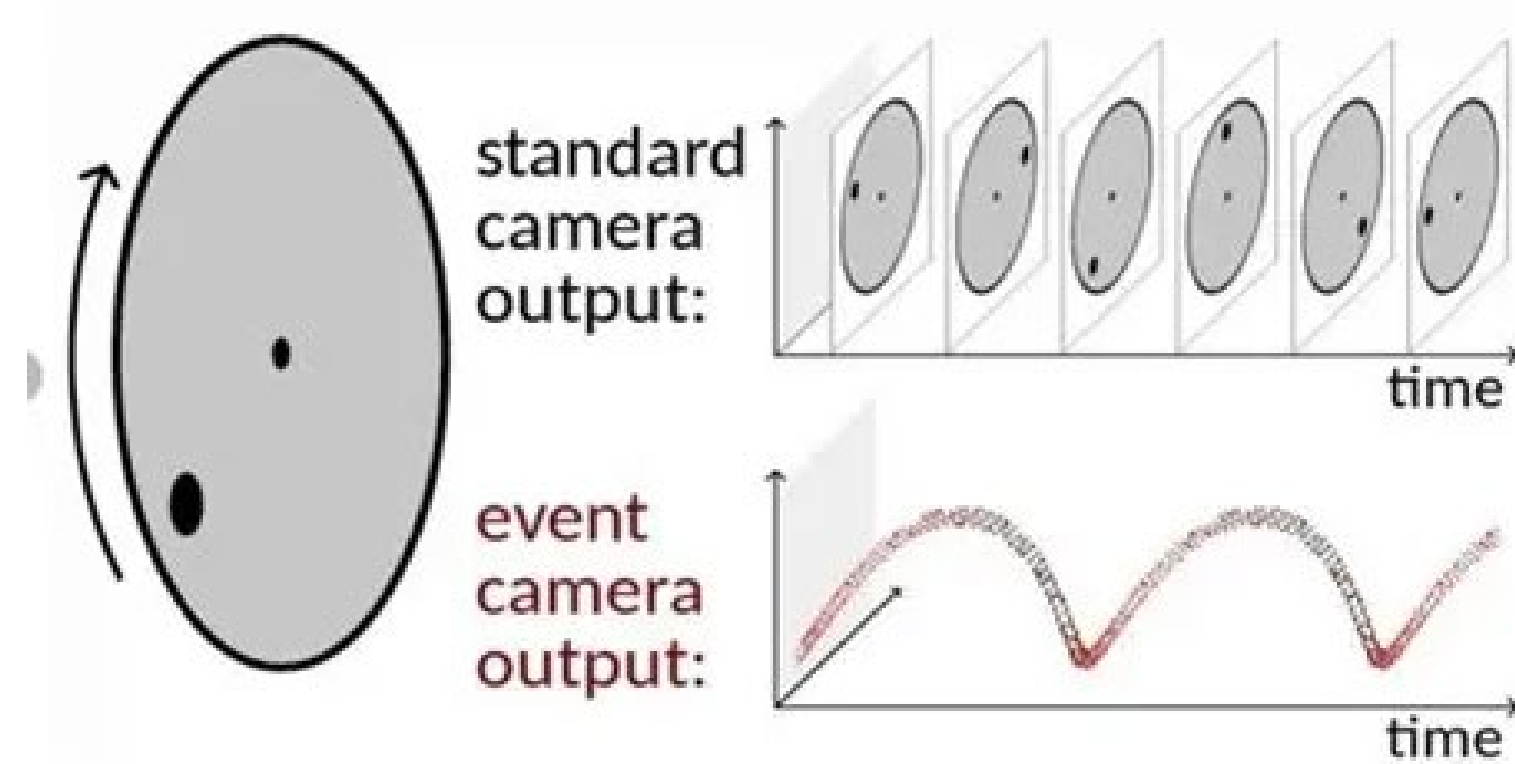


Figure 1. Output of RGB vs event camera

Background

Raw events are **sparse and asynchronous**, requiring structured formats to make the data digestible. We experiment with representing events as **heat kernels** that diffuse over time, allowing events to have a temporal component

Event Acquisition

RAW events from the highcontrastline EEPR dataset are grouped using a temporal diffusion window and organized into ON/OFF channels.

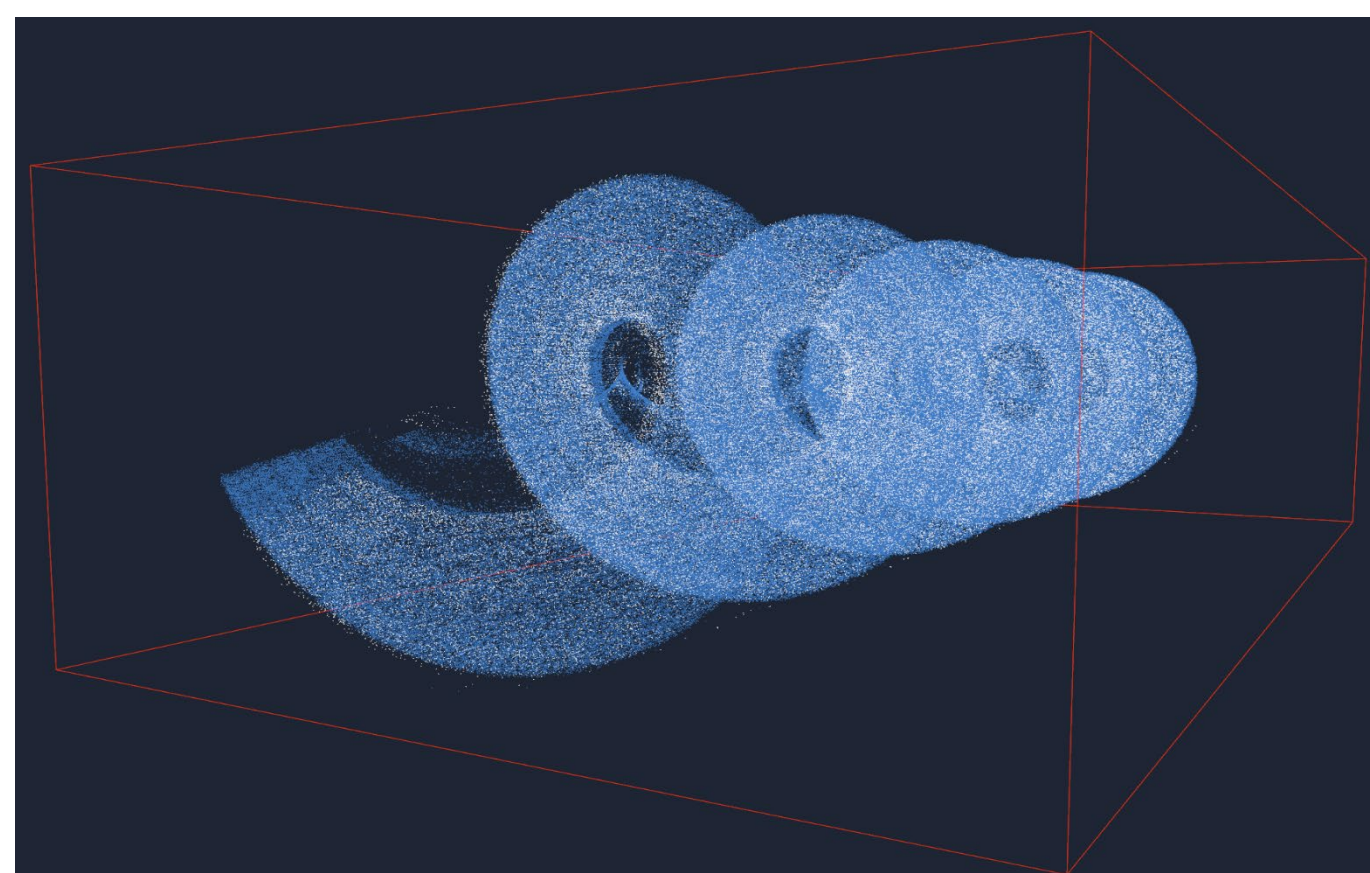


Figure 2. 250ms of the data visualised in XYTime

Representation & Analysis

A heatmap is constructed from recent events, from which gradients and the Laplacian are computed to extract temporal patterns and estimate motion frequency. How the heatmap is generated is discussed in experimental setup.

Experimental Setup

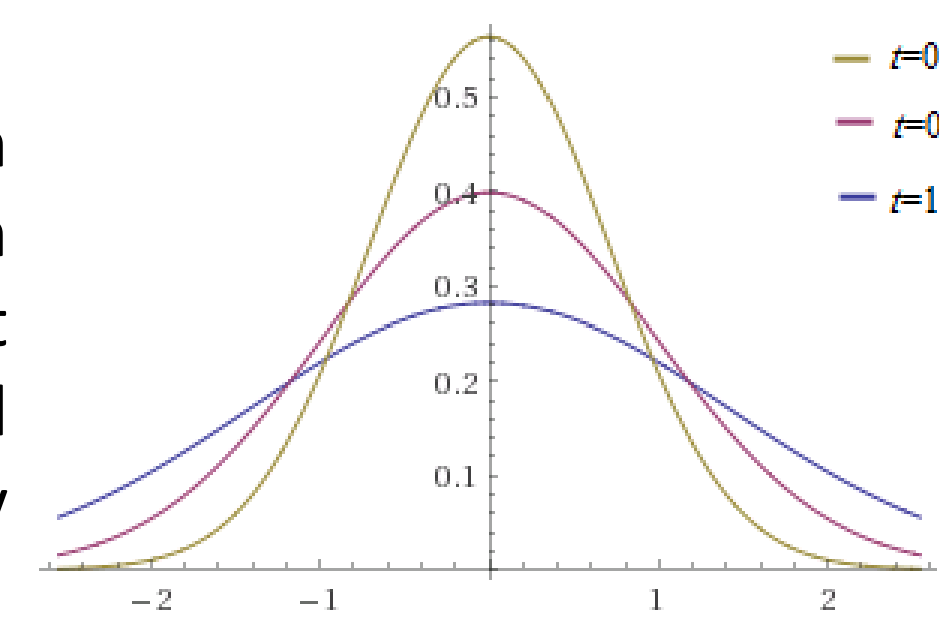
The heat equation is

$$\frac{\partial H}{\partial t} = \alpha \nabla^2 H$$

Which describe how heat (H) spreads through a medium over time. $\frac{\partial H}{\partial t}$ represents the rate of change of the heatmap intensity over time, α is the constant that controls the diffusion rate and $\nabla^2 H$ is the **Laplacian** of heat, which measures the rate at which heat spreads throughout space. For a point source of heat originating from point (x_0, y_0) at time t_0 , the magnitude of diffused heat at (x, y) and time t would be

$$H(x, y, t) = \frac{1}{4\pi\alpha(t - t_0)} \exp\left(-\frac{(x - x_0)^2 + (y - y_0)^2}{4\alpha(t - t_0)}\right)$$

If event activity is modeled as a heat-diffusion field, the **Laplacian** highlights motion-driven structural changes in the signal. Its dominant frequency can track the underlying physical oscillation, supporting efficient frequency estimation from event data



- Events are modeled as point sources, constructed by superposition of Green's-function responses over a causal time window.
- Derived fields $|\nabla H|$ and $\nabla^2 H$ emphasize boundaries and suppress slow baseline structure, enabling reliable frequency detection at all α where raw heat alone requires $\alpha \approx 100,000$. Heat diffusion sees a significant decline in performance in the range $100 < \alpha < 100,000$.
- The diffusion coefficient α controls a bias-variance tradeoff: too low leaves the field spatially fragmented; too high blurs the oscillation, reliable estimation spans $\alpha \approx 1,000$ to $10,000$.
- Frequency is estimated via FFT per pixel, cross-checked with inter-peak spacing and autocorrelation; binned raw event counts serve as an α -free reference to confirm physical detections.

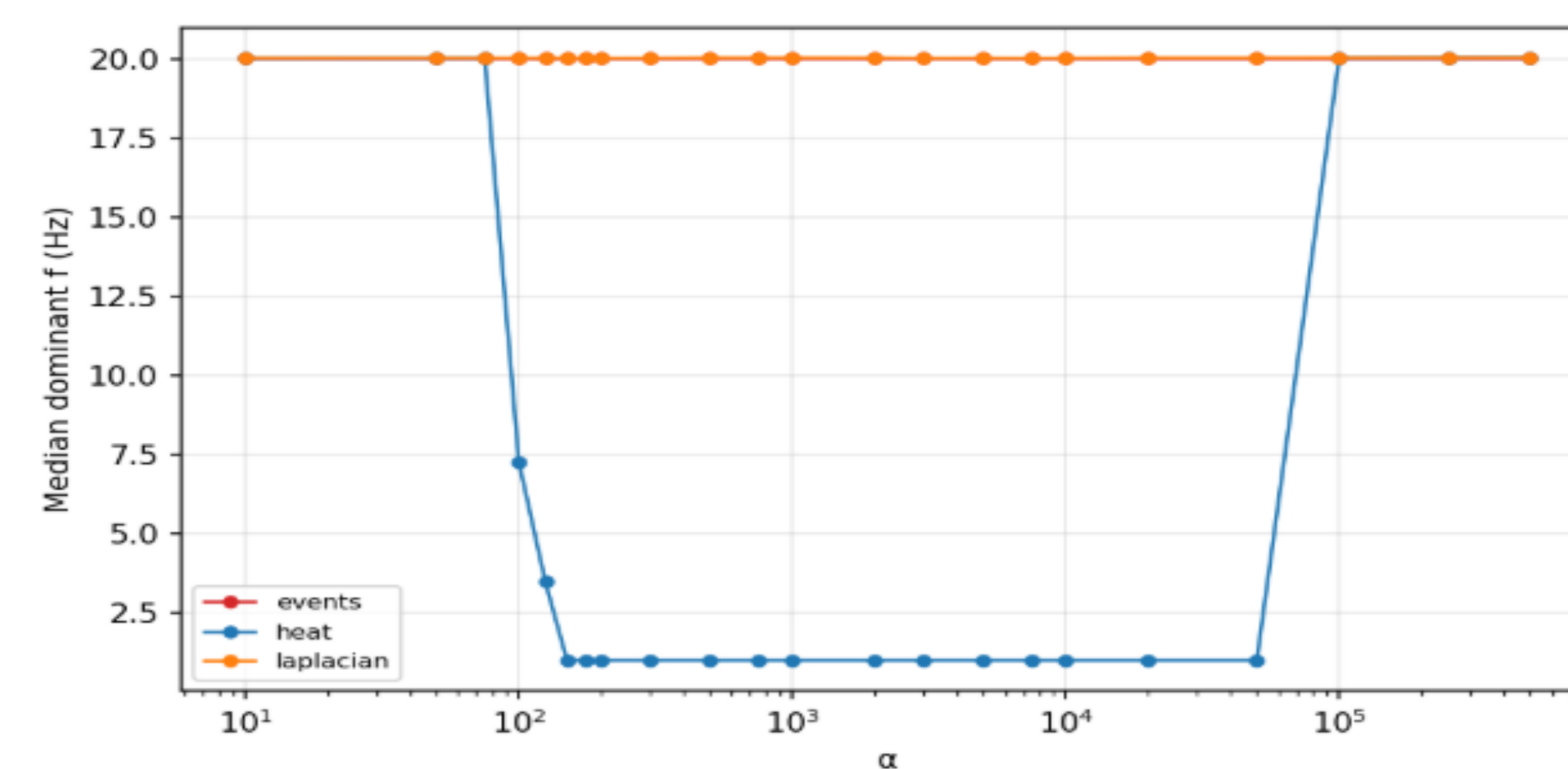
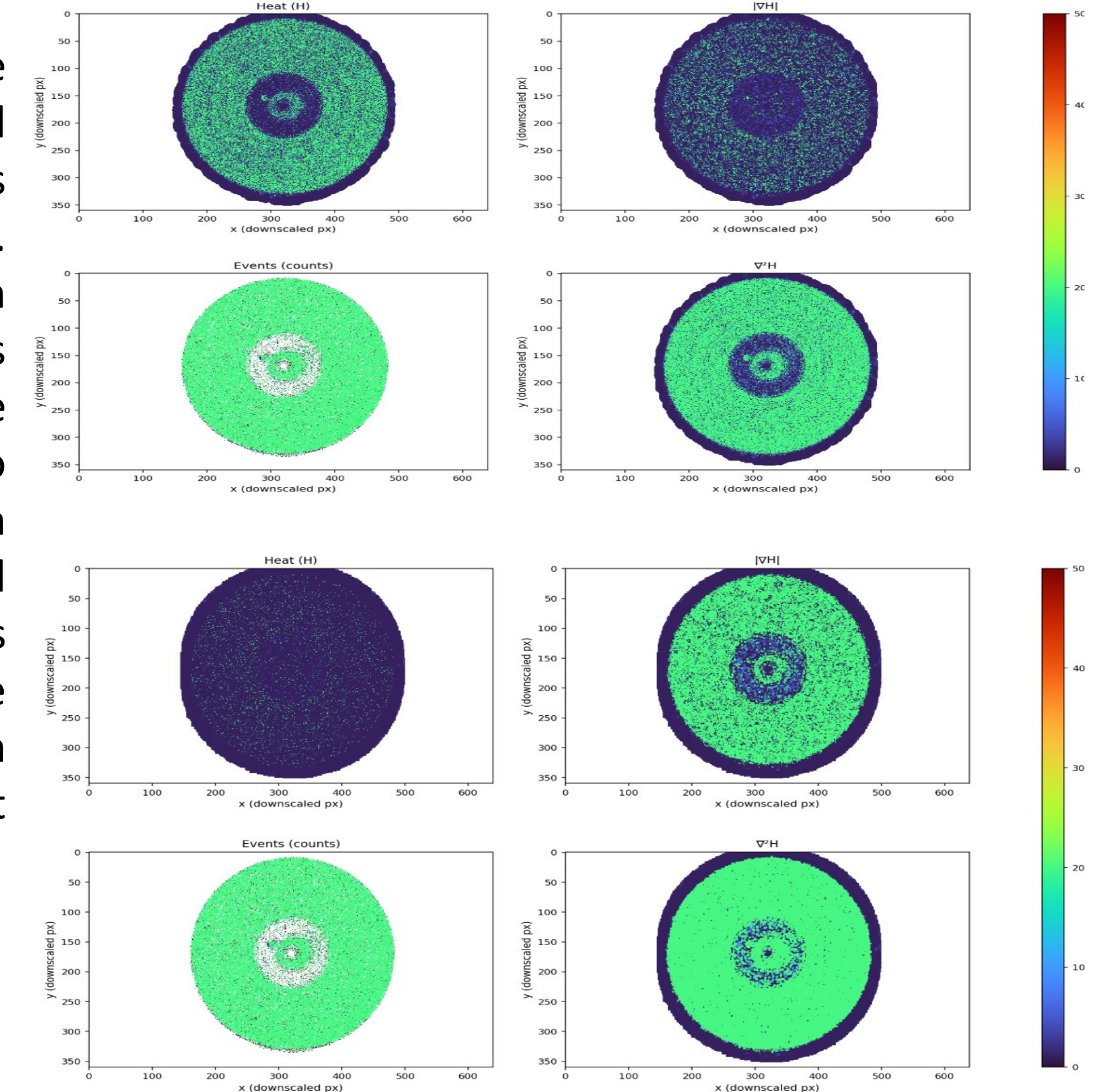


Figure 3. Effect of diffusion coefficient on frequency estimation across event, heat, and Laplacian representation

Note that: Per-frame heatmaps are generated from aligned events using kernel superposition. ROI signal peaks are then mapped to period and frequency.

Experimental Results

These panels compare event, heat, gradient, and Laplacian representations across diffusion settings. Raw event counts remain unchanged with α . Heat is fragmented but accurate near $\alpha=50$, then collapses to about 1.01Hz at higher α . In contrast, Laplacian and gradient-based estimates remain accurate across the full α range, with Laplacian improving until about $\alpha=10,000$.



Conclusions

- **Raw-event baseline:** Binned event counts are α -independent and consistently preserve the true **20 Hz** frequency across the full sweep.
- **Diffusion benefit and limit:** Plain heat collapses to about **1 Hz** for $\alpha \in [100, 100,000]$ due to baseline dominance, then recovers only at very large α when spatial coverage saturates.
- **Derivative advantage:** The Laplacian preserves the correct **20 Hz** estimate across the full α range and remains robust where plain heat fails, supporting baseline suppression as the key advantage.

With these findings, we can conclude:

- Diffusion heatmaps enable accurate frequency estimation from sparse data; Laplacian provides the best spatial-temporal balance.
- This framework opens a path toward real-time periodic motion analysis in rotating machinery inspection, biomechanics, and high-speed robotics.

References

1. Kolář, J., Špetlík, R., and Matas, J., "EEPPR: Event-based Estimation of Periodic Phenomena Frequency using 3D Correlation," *Proceedings of the 17th International Conference on Machine Vision*, 2024.
2. Zheng, X., Liu, Y., Lu, Y., Hua, T., Pan, T., Zhang, W., Tao, D., and Wang, L., "Deep Learning for Event-based Vision: A Comprehensive Survey and Benchmarks," *arXiv:2302.08890*, 2024.
3. Perot, E., de Tournemire, B., Nitti, D., Masci, J., and Sironi, A., "Learning to Detect Objects with a 1 Megapixel Event Camera," *Advances in Neural Information Processing Systems*, 2020, Vol. 33, pp. 16639-16652.